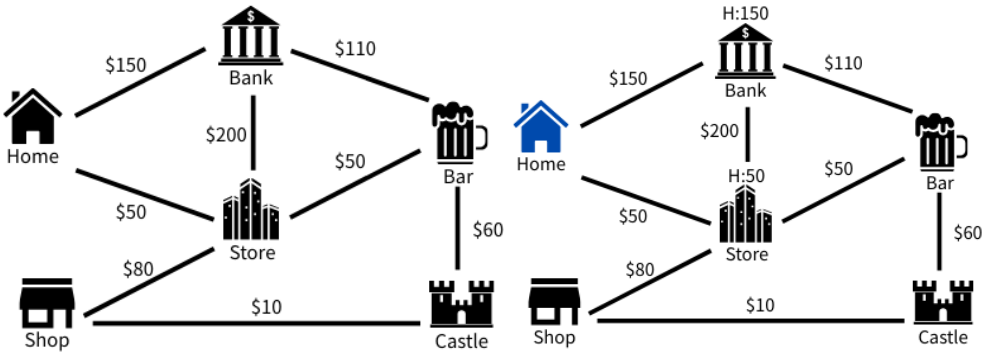
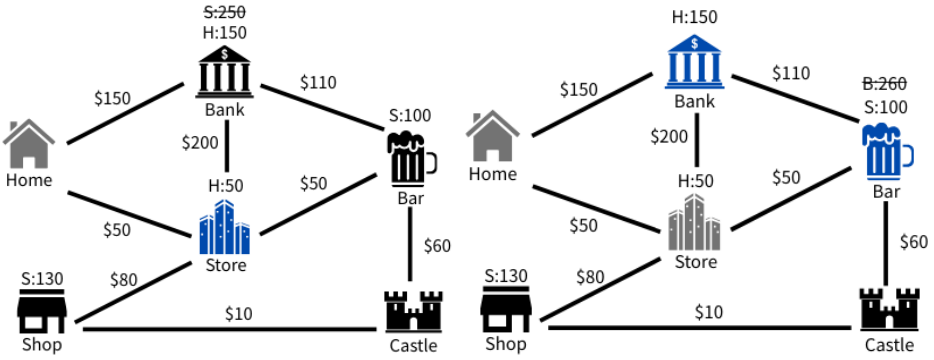
Units 10, 11

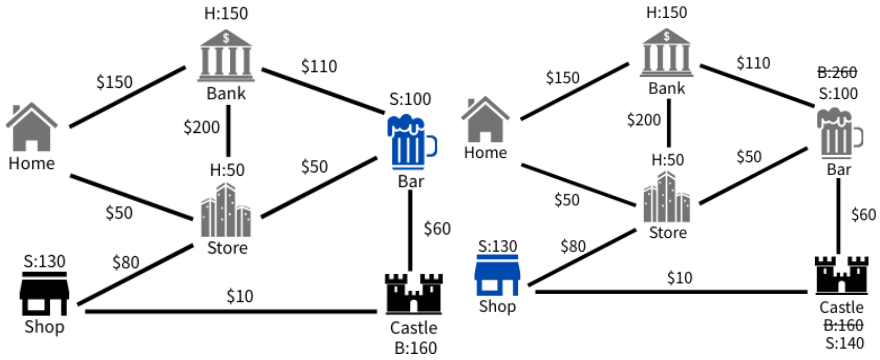
Algorithms

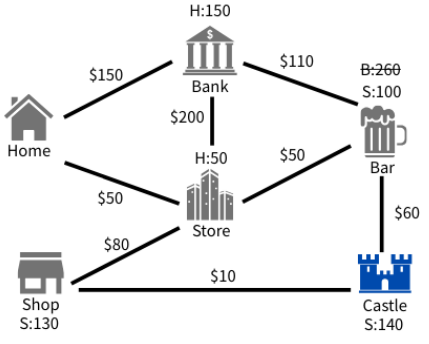
1.

$$Shortest_{HC} = \min_{i \in \{Home, Shop, Bank, Bar, Store, Castle\}} (Shortest_{Hi} + Cost_{iC})$$









Ans: Home→Store→Shop→Castle,

The minimum transportation cost from Home to Castle is $140.

2.

Ans:

$$\Theta(1), \Theta(\log n) = \Theta(\log(n^2)), \Theta((\log n)^2), \Theta(\sqrt{n}) = \Theta\left(\frac{\sqrt{n^2+1}}{\sqrt{n}}\right), \Theta\left(\frac{n}{\log n}\right), \Theta(n) = \Theta(n + \log n), \Theta(n^2)$$

3.

$m[i, i] = 0$ for all i

- two matrices at a time:

$m[1,2] = 0 + 0 + 8 \times 5 \times 2 = 80$

$m[2,3] = 0 + 0 + 5 \times 2 \times 6 = 60$

$m[3,4] = 0 + 0 + 2 \times 6 \times 3 = 36$

- three matrices at a time:

$m[1,3] = min\{m[1,1] + m[2,3] + 8 \times 5 \times 6, m[1,2] + m[3,3] + 8 \times 2 \times 6\}$

$= min\{0 + 60 + 8 \times 5 \times 6, 80 + 0 + 8 \times 2 \times 6\} = min\{300,176\} = 176$

$m[2,4] = min\{m[2,2] + m[3,4] + 5 \times 2 \times 3, m[2,3] + m[4,4] + 5 \times 6 \times 3\}$

$= min\{0 + 36 + 5 \times 2 \times 3, 60 + 0 + 5 \times 6 \times 3\} = min\{66,150\} = 66$

- all matrices together:

$m[1,4] = min\{m[1,1] + m[2,4] + 8 \times 5 \times 3, m[1,2] + m[3,4] + 8 \times 2 \times 3, m[1,3] + m[4,4] + 8 \times 6 \times 3\}$

$= \min\{0 + 66 + 8 \times 5 \times 3, 80 + 36 + 8 \times 2 \times 3, 176 + 0 + 8 \times 6 \times 3\}$

$= min\{186, 164, 320\} = 164$

Ans: $(A_1A_2)(A_3A_4), 164$