# The language of set theory

- Sets and elements
  - $S_1 = \{a, b, c, d\}$, $S_2 = \{a, b, e\}$.
  - $S_1$ is a **set**. $S_2$ is a **set**.
  - $a$ is an **element** *of* $S_1$ .

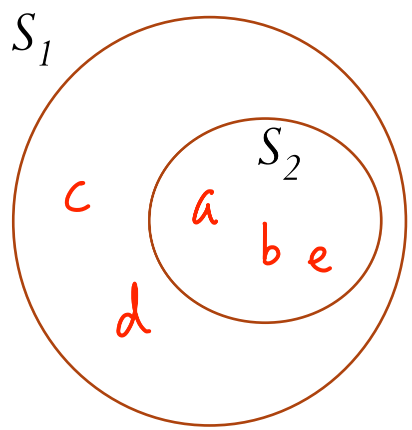
- $\in$: "is in", $\notin$: "is not in"
  - $a \in S_{1,}\, a \in S_2$
  - e $\notin S_1, e \in S_2$

- Empty set
  - $S_3 = \{\}$. $\phi$

# The language of set theory

- Let $\boldsymbol{S}_1 = \{a, b, c, d, e\}$, $\boldsymbol{S}_2 = \{a, b, e\}$.

  $S_1 = \{b, a, d, e, c\}$ $S_3 = \{a, e, c, d, b\}$




- $\subset$ : "subset" (or $\subseteq$)

  $\mathcal{S}_2 \subset \mathcal{S}_1$ means
  $\forall x \in \mathcal{S}_2$, $x$ is also $\in \mathcal{S}_1$.

- $=$: "equal sets"
  - $\mathcal{A} = \mathcal{B}$ means $\forall x \in \mathcal{A}$, $x$ is also $\in \mathcal{B}$ and; $\forall y \in \mathcal{B}$, $y$ is also $\in \mathcal{A}$.
  - In other words, $\mathcal{A} \subset \mathcal{B}$ and $\mathcal{B} \subset \mathcal{A}$.

- Let $\boldsymbol{S}_3 = \{e, b, d, c, a\}$. Then $\boldsymbol{S}_1 = \boldsymbol{S}_3$.

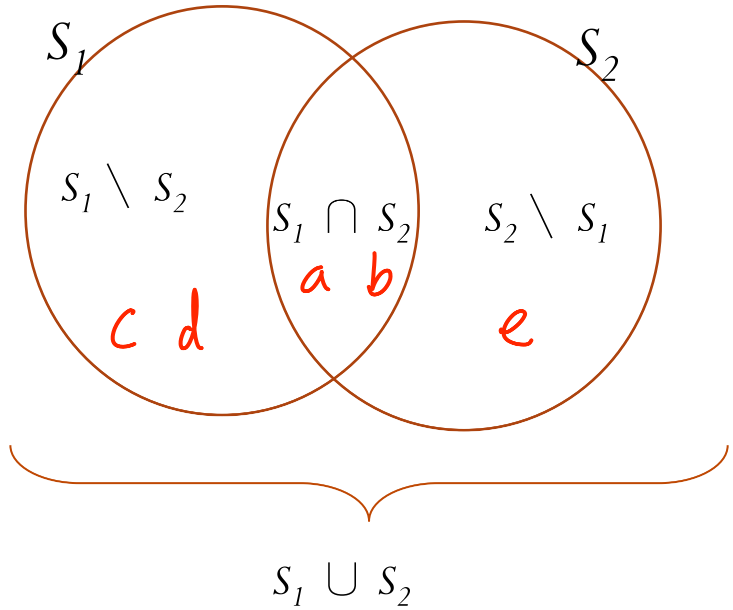
# The language of set theory

- Let $S_1 = \{a, b, c, d\}$, $S_2 = \{a, b, e\}$.

- $\cup$ : "union set"
  - $S_1 \cup S_2 = \{a, b, c, d, e\}$

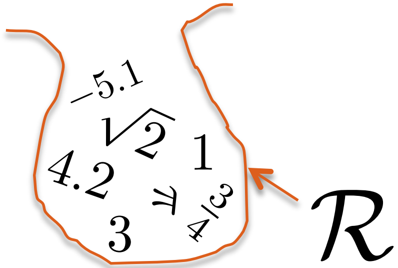
- $\cap$ : "intersection set"
  - $S_1 \cap S_2 = \{a, b\}$

- $\setminus$: "difference set"
  - $S_1 \setminus S_2 = \{c, d\}$
  - $S_2 \setminus S_1 = \{e\}$

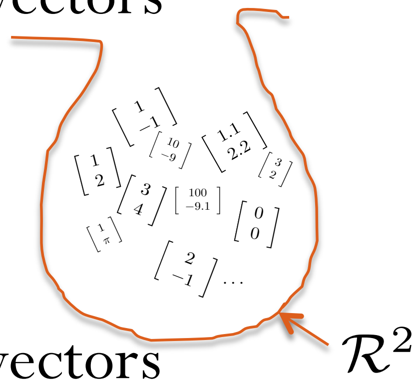


# Sets that contain infinite elements

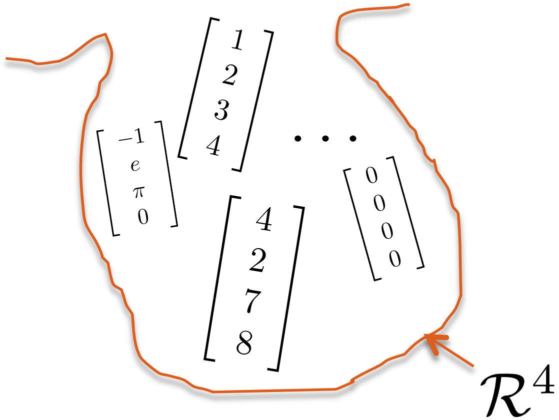
- $\mathcal{R}$ is the set that contain all real numbers.




- $\mathcal{R}^2$ is the set the contain all real-valued column vectors whose has two components.
  - $$\mathcal{R}^2 = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : x_1, x_2 \in \mathcal{R} \right\}$$



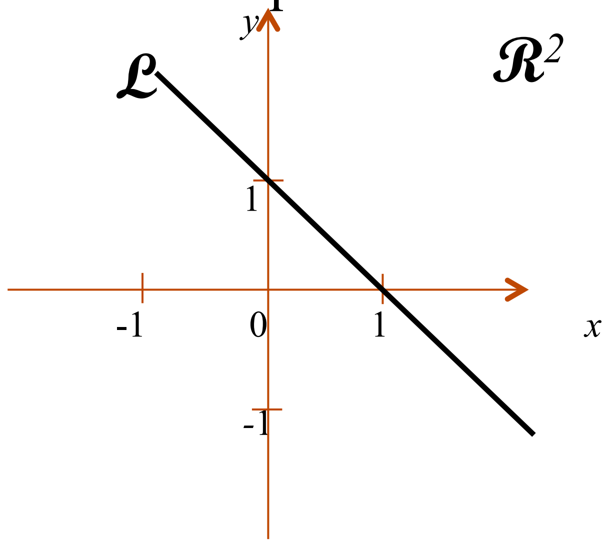

- $\mathcal{R}^n$ is the set the contain all real-valued column vectors whose number of components is $n$.
  - $$\mathcal{R}^n = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} : x_1, x_2, \cdots, x_n \in \mathcal{R} \right\}$$



# Sets that contain infinite elements

- Let $\mathcal{L}$ denote the line on the xy-plane whose equation is

$$x + y = 1.$$



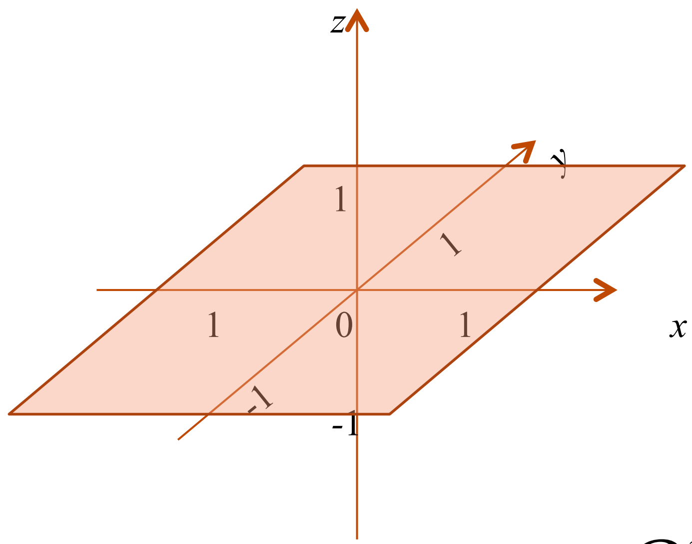

- Then $\mathcal{L}$ is a **subset** of $\mathcal{R}^2$ and

$$\mathcal{L} = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : x_1, x_2 \in \mathcal{R}, x_1 + x_2 = 1 \right\}$$

$$= \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x, y \in R, \; x + y = 1 \right\}$$

# Questions

$R^2 \not\subset R^3$




- Is $\mathcal{R}^2$ a **subset** of $\mathcal{R}^3$ ?

$R^2 = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \; x, y \in R \right\}$

$R^3 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; x, y, z \in R \right\}$

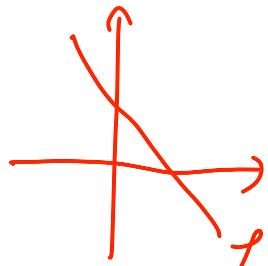
- Is the plane determined by equation "z = 0" a **subset** of $\mathcal{R}^3$ ?

$$\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x, y, z \in \mathcal{R}, z = 0 \right\} \checkmark$$

- How do we express the plane "z = 0" as a set?

$\left\{ x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} : x, y \in R \right\} \checkmark$

# Questions



$\mathcal{L}_1 = \mathcal{L}_2,$
$\mathcal{L}_3, \mathcal{L}_4$

- What is the difference between the following two sets?

$$\mathcal{L}_1 = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : x_1, x_2 \in \mathcal{R}, x_1 + x_2 = 1 \right\}$$

$$\mathcal{L}_2 = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x, y \in \mathcal{R}, x + y = 1 \right\}$$

$\mathcal{L}_1 = \mathcal{L}_2 = \mathcal{L}_3$
$= \mathcal{L}_4$

- How about
$$\mathcal{L}_3 = \{\mathbf{v} : \mathbf{v} \in \mathcal{R}^2, A\mathbf{v} = 1 \text{ where } A = [\,1 \quad 1\,]\}$$
$$\mathcal{L}_4 = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \end{bmatrix} : t \in \mathcal{R} \right\}$$
?

$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \in R^2$

- Is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ a **subset** of $\mathcal{R}^2$? (i.e., is $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \subset \mathcal{R}^2$ true?) No
- Is $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ a **subset** of $\mathcal{R}^2$? Yes